Introduction to Quantum Teleportation

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Quantum teleportation, a technique for transferring quantum states from one place to another arbitrarily distant place without crossing the intervening space, was first conceptualized by Bennett et al. in 1993. Subsequent experiments, such as Boschi et al. in 1998, have confirmed their predictions by successfully teleporting the polarization states of photons. The theory has since been expanded to include systems of continuous variables, and experiments have succeeded in teleporting the electronic states of atoms.

INTRODUCTION

Teleportation, a term originally coined by science fiction, refers to a hypothetical technology that can transport objects (or, eventually, people) nearly instantaneously from one location to another without sending the object through the intervening space. Classically, one might approach this problem by attempting to record the states of all the particles constituting the object to be teleported; that information could then be transmitted to a distant receiver and used to reconstitute the object out of raw materials available at the receiver. For many years this approach to teleportation was considered implausible because of quantum mechanical concerns. For instance, a teleportation device must somehow record the precise positions and momenta of all atoms in an object in order to reconstruct the object on the other side. This simultaneous measurement of non-commuting observables is forbidden by the uncertainty principle. A more fundamental problem exists, though, which is evident even in situations where the uncertainty principle is not directly applicable. To illustrate this point, consider the teleportation of a two-state system, such as the polarization state of a single photon:

$$|\varphi\rangle = \alpha\,|H\rangle + \beta\,|V\rangle \qquad \text{(where } |\alpha|^2 + |\beta|^2 = 1\text{)} \tag{1}$$

Here, $|H\rangle$ ($|V\rangle$) represents horizontal (vertical) polarization. The polarization state of the photon is fully described by the complex numbers α and β, but attempting to measure the polarization simply results in the collapse of the state into $|H\rangle$, $|V\rangle$, or some linear combination of those bases (as in the case of a polarizing beamsplitter set at a 45° angle). This collapse generally yields only one bit of information about the state, which is inadequate because the numbers α and β contain a literally infinite amount of information. Only if many copies of the state $|\varphi\rangle$ exist can repeated measurements place statistical bounds on the values of α and β. This loophole suggests a method of determining the state (albeit an arduous one): take the state to be teleported and copy it many times, then measure each copy separately to produce an estimate of α and β whose accuracy is limited only by the number of copies made. Unfortunately, the no-cloning theorem[1] from quantum information theory strictly prohibits copying unknown quantum states, which would seem to suggest that teleporting even this simple system is impossible.

THEORY

In 1993, a solution to this dilemma was proposed[2] by six co-authors, collectively known as BBCJPW. The BBCJPW protocol uses a property of quantum mechanics called "entanglement" to teleport a two-state system. Quantum entanglement was first discovered by Einstein, Podolsky and Rosen in the famous 1935 "EPR" paper[3], though Einstein regarded it as an unphysical prediction which suggested the existence of "hidden variables" that he believed would form the basis of a more complete theory which would supersede quantum mechanics.

A pair of particles is said to be entangled if the quantum state of the pair cannot be separated into two factors, each containing terms related to a single particle. In other words, it's impossible to fully describe one particle without referring to the other particle. For instance, the following singlet state is entangled:

$$\left|\Psi_{12}^{(-)}\right\rangle = \frac{1}{\sqrt{2}}\left(|H_1\rangle\,|V_2\rangle - |V_1\rangle\,|H_2\rangle\right) \tag{2}$$

The distinctive feature of quantum entanglement is that measurements performed on one particle are correlated with the other particle's measurements regardless of their separation distance. For example, multiplying eq. 2 on the left by $\langle H_1|$ results in particle 2's collapse into a state of definite vertical polarization:

$$\left\langle H_1 \middle| \Psi_{12}^{(-)} \right\rangle = \frac{1}{\sqrt{2}} |V_2\rangle \tag{3}$$

Physically, this means that if particle 1 is found to be horizontally polarized, a measurement on particle 2 is certain to reveal that it is vertically polarized, even if the second measurement is performed so soon after the first that special relativity prevents particle 2 from "knowing" about the results of particle 1's measurement. At first glance, this phenomenon seems like a clear example of faster-than-light correlations, but Einstein's idea of "hidden variables" suggests a more radical explanation: there really is no such thing as a quantum superposition. In other words, only one term in eq. 2 is "real", the other term arises from the incompleteness of quantum theory. If the first particle was always horizontally polarized, despite our inability to describe this "hidden reality" with quantum mechanics, then the fact that the second one is vertically polarized is just a consequence of the conservation of angular momentum— it involves no "spooky action at a distance."

Bell considered[4] the possibility of hidden variables three decades later, and found that quantum mechanics and "local hidden variable theories" predict different results under certain circumstances. Local hidden variable theories assert that the state of a particle is controlled by a set of variables that depend only on events in the past light cone of that particle. The departure from quantum mechanics arises when, instead of measuring the polarization of each entangled particle in the manner of eq. 3, each particle is measured in a randomly oriented basis. This deviation has come to be known as Bell's Inequality, and experiments[5,6] have since confirmed the predictions of quantum mechanics. Bell's Inequality is now used in an experimental context to verify that two particles are in fact entangled.

The BBCJPW teleportation protocol requires that the sender (Alice) and the receiver

(Bob) have previously shared two halves of an entangled state, as described in eq. 2. Particle 1 (2) is Bob's (Alice's) half of the EPR pair, and particle 3, which Alice wishes to teleport to Bob, is in the state $|\varphi_3\rangle$ as defined in eq. 1. Particle 3 and the entangled pair are initially in the pure product state

$$\left|\Psi_{12}^{(-)}\right\rangle |\varphi_3\rangle = \frac{1}{\sqrt{2}}\left[\left(|H_1\rangle |V_2\rangle - |V_1\rangle |H_2\rangle\right)\left(\alpha |H_3\rangle + \beta |V_3\rangle\right)\right] \tag{4}$$

Alice now performs a simultaneous complete measurement on the particles in her possession (particles 2 and 3) in the Bell basis which consists of the basis vectors

$$\left|\Psi_{23}^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|H_2\rangle |V_3\rangle \pm |V_2\rangle |H_3\rangle\right) \tag{5a}$$

$$\left|\Phi_{23}^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|H_2\rangle |H_3\rangle \pm |V_2\rangle |V_3\rangle\right) \tag{5b}$$

This measurement has the effect of collapsing the state given in eq. 4 onto one of the four Bell basis vectors given in eq. 5. Note that the Bell basis is orthonormal for particles 2 and 3, and can thus be inverted to obtain

$$|H_2\rangle |V_3\rangle = \frac{1}{\sqrt{2}}\left(\left|\Psi_{23}^{(+)}\right\rangle + \left|\Psi_{23}^{(-)}\right\rangle\right) \tag{6a}$$

$$|V_2\rangle |H_3\rangle = \frac{1}{\sqrt{2}}\left(\left|\Psi_{23}^{(+)}\right\rangle - \left|\Psi_{23}^{(-)}\right\rangle\right) \tag{6b}$$

$$|H_2\rangle |H_3\rangle = \frac{1}{\sqrt{2}}\left(\left|\Phi_{23}^{(+)}\right\rangle + \left|\Phi_{23}^{(-)}\right\rangle\right) \tag{6c}$$

$$|V_2\rangle |V_3\rangle = \frac{1}{\sqrt{2}}\left(\left|\Phi_{23}^{(+)}\right\rangle - \left|\Phi_{23}^{(-)}\right\rangle\right) \tag{6d}$$

In order to express the full state of all three particles in the new basis, substitute eq. 6 into eq. 4 to obtain

$$\left|\Psi_{12}^{(-)}\right\rangle |\varphi_3\rangle = \frac{1}{2}\left[\begin{array}{c} \left(\alpha |H_1\rangle - \beta |V_1\rangle\right)\left|\Psi_{23}^{(+)}\right\rangle + \left(-\alpha |H_1\rangle - \beta |V_1\rangle\right)\left|\Psi_{23}^{(-)}\right\rangle \\ + \left(-\alpha |V_1\rangle + \beta |H_1\rangle\right)\left|\Phi_{23}^{(+)}\right\rangle + \left(-\alpha |V_1\rangle - \beta |H_1\rangle\right)\left|\Phi_{23}^{(-)}\right\rangle \end{array}\right] \tag{7}$$

Notice that four factors in eq. 7 look similar to the original state $|\varphi_3\rangle$, except that the relevant factors α and β now multiply kets describing particle 1. In fact, if we identify $\alpha\,|H_1\rangle + \beta\,|V_1\rangle$ as $|\varphi_1\rangle$, eq. 7 takes on a more suggestive form

$$\left|\Psi_{12}^{(-)}\right\rangle|\varphi_3\rangle = \frac{1}{2}\left[\begin{array}{l} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}|\varphi_1\rangle\left|\Psi_{23}^{(+)}\right\rangle + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}|\varphi_1\rangle\left|\Psi_{23}^{(-)}\right\rangle \\ + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}|\varphi_1\rangle\left|\Phi_{23}^{(+)}\right\rangle + \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}|\varphi_1\rangle\left|\Phi_{23}^{(-)}\right\rangle \end{array}\right] \tag{8}$$

Alice's measurement collapses the state onto one of these four terms. If Bob knows the result of Alice's measurement, he could multiply his state by the inverse of the relevant matrix in eq. 8, thus leaving him with the state $|\varphi_1\rangle$. This new state is identical to Alice's state $|\varphi_3\rangle$ except it is in Bob's lab which is arbitrarily far away from Alice. The fact that Bob needs to know which Bell basis state was measured by Alice limits the maximum speed at which teleportation can be completed to the speed of light. Notice that the teleportation process does not transport energy or matter, it simply transfers quantum states from one particle to another. Calling this state transfer "teleportation" is valid only for indistinguishable particles, where particles in identical states are impossible to tell apart even in principle. Also, the no-cloning theorem is not violated because the original particle is no longer in the state $|\varphi_3\rangle$, it is now in one of the Bell basis states along with particle 2.

EXPERIMENT

The first experimental demonstration[7] of quantum teleportation of photons was performed in 1997, but the setup was only able to distinguish one of the four Bell basis states in eq. 5, so teleportation was only successful with at most 25% of the incoming photons. In 1998, a team led by D. Boschi overcame[8] this limitation, with the caveat that the state to be teleported was not an external photon separate from the EPR pair. Instead of using polarization entanglement, the Boschi experiment used **k**-vector (or path) entanglement, and imprinted the state to be teleported on the polarization of the photon that was sent to Alice. This state cannot be determined by Alice, and in principle the experiment could be extended to teleport an external photon's polarization state by swapping its polarization with that of Alice's EPR photon. Their experimental setup is shown in figure 1.

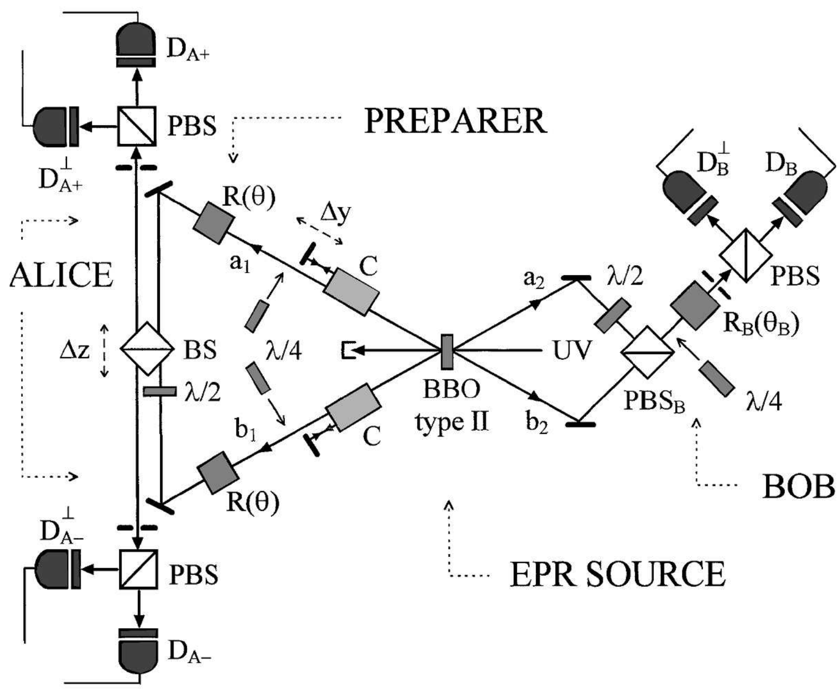

Fig. 1 - Boschi teleportation experiment setup (adapted from [8])

The EPR pair is generated by type II degenerate parametric down-conversion[9] in the β-barium borate (BBO) crystal in the center of figure 1. In other words, the ultraviolet pumping laser (UV) shines through the BBO crystal where nonlinearities in the crystal's electric susceptibility cause some of these photons to split into two photons while conserving energy and momentum. By cutting the crystal to the appropriate size and shape, the down-converted photons leave the crystal in different directions (paths a_1 and b_1 in figure 1) with opposite polarizations, in the state given by eq. 9.

$$\left|\Psi_{12}^{(+)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|H_1\right\rangle\left|V_2\right\rangle + \left|V_1\right\rangle\left|H_2\right\rangle\right) \tag{9}$$

This polarization entanglement is then converted to path entanglement by placing calcite crystals (C) in the paths of both outgoing photons. Calcite is birefringent, so photons with different polarizations emerge from the calcite in distinct paths. After each calcite crystal, a mirror blocks the path taken by the horizontal polarization, reflecting its amplitude back to Bob through both the calcite and BBO crystals. This converts the initial state in eq. 9 into the following state:

$$\frac{1}{\sqrt{2}}\left(|a_1\rangle |a_2\rangle + |b_1\rangle |b_2\rangle\right) |V_1\rangle |H_2\rangle \tag{10}$$

Particle 1(2) now refers to Alice's (Bob's) photon. The kets $|a_1\rangle$,$|a_2\rangle$,$|b_1\rangle$, and $|b_2\rangle$ refer to the paths as labelled in figure 1. Notice that the polarizations are no longer entangled because the particle heading towards Alice (Bob) is definitely vertically (horizontally) polarized. The entanglement now involves the path taken by each photon- if Alice's photon travels through path a_1 (b_1) then Bob's photon is certain to be in path a_2 (b_2). At this point the "Preparer" uses a set of Fresnel rhomb polarization rotators (R) to rotate the polarization of Alice's photon (acting identically in both paths) through an angle θ. The polarization of Alice's photon is now in a state given by eq. 1 where $\alpha = \sin(\theta)$ and $\beta = \cos(\theta)$. The complete state is now

$$\frac{1}{\sqrt{2}}\left(|a_1\rangle |a_2\rangle + |b_1\rangle |b_2\rangle\right) \left(\alpha |H_1\rangle + \beta |V_1\rangle\right) |H_2\rangle \tag{11}$$

In analogy with the original BBCJPW protocol, Alice performs a complete measurement in a new basis given by

$$\left|c^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|a_1\rangle |V_1\rangle \pm |b_1\rangle |H_1\rangle\right) \tag{12a}$$

$$\left|d^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|a_1\rangle |H_1\rangle \pm |b_1\rangle |V_1\rangle\right) \tag{12b}$$

Once again, this basis is orthonormal, and can be inverted and substituted into eq. 11, which results in

$$\frac{1}{2}\left[\begin{array}{c} (\alpha\left|a_2\right\rangle+\beta\left|b_2\right\rangle)\left|H_2\right\rangle\left|c^{(+)}\right\rangle+(\alpha\left|a_2\right\rangle-\beta\left|b_2\right\rangle)\left|H_2\right\rangle\left|c^{(-)}\right\rangle \\ +(\beta\left|a_2\right\rangle+\alpha\left|b_2\right\rangle)\left|H_2\right\rangle\left|d^{(+)}\right\rangle+(\beta\left|a_2\right\rangle-\alpha\left|b_2\right\rangle)\left|H_2\right\rangle\left|d^{(-)}\right\rangle \end{array}\right] \tag{13}$$

Now Alice has to measure her photon in such a way that all four bases in eq. 12 are distinguishable. To do this, she first rotates the polarization of path b_1 by 90° using a half-wave plate, which changes $|b_1\rangle|H_1\rangle$ to $|b_1\rangle|V_1\rangle$ and $|b_1\rangle|V_1\rangle$ to -$|b_1\rangle|H_1\rangle$. This rotation effectively applies the following transformation to the basis in eq. 12.

$$\left|c^{(\pm)}\right\rangle \longrightarrow \frac{1}{\sqrt{2}}(|a_1\rangle \pm |b_1\rangle)|V_1\rangle \tag{14a}$$

$$\left|d^{(\pm)}\right\rangle \longrightarrow \frac{1}{\sqrt{2}}(|a_1\rangle \mp |b_1\rangle)|H_1\rangle \tag{14b}$$

Both paths then hit a non-polarizing 50:50 beam splitter (BS) and are either reflected or transmitted to the two polarizing beamsplitters (PBS), each with its own set of detectors. The detectors labeled $D^{\perp}_{A\pm}$ ($D_{A\pm}$) are arranged so they detect only vertically (horizontally) polarized light. Because of the rotation applied to path b_1, $\left|c^{(\pm)}\right\rangle$ and $\left|d^{(\pm)}\right\rangle$ are distinguishable based on their polarization; a detected photon (or "click") at detector $D^{\perp}_{A\pm}$ corresponds to detection of $\left|c^{(\pm)}\right\rangle$ while a click at detector $D_{A\pm}$ corresponds to detection of $\left|d^{(\mp)}\right\rangle$. Distinguishing between $\left|c^{(+)}\right\rangle$ and $\left|c^{(-)}\right\rangle$ (or equivalently between $\left|d^{(+)}\right\rangle$ and $\left|d^{(-)}\right\rangle$) is accomplished by positioning BS so that the state $\frac{1}{\sqrt{2}}(|a_1\rangle+|b_1\rangle)$ interferes at BS in such a way as to send it to detectors $D^{\perp}_{A+}$ and D_{A+}, and vice-versa. In this manner all four Bell states can be distinguished from each other based on which of the four detectors clicks.

Alice's measurement thus collapses the state in eq. 13 into a single term as intended, but the factor describing Bob's photon is in superposition of paths instead of the desired polarization superposition. To produce a polarization superposition, Bob inserts a half-wave plate in path a_2 to rotate its polarization from horizontal to vertical. Both paths a_2 and b_2 then hit a polarizing beamsplitter (PBS_B) which reflects (transmits) amplitude from path

a_2 (b_2) so that the amplitude from each path emerges in a single direction. The complete state is now

$$\frac{1}{2}\left[\begin{array}{c} (\beta\left|H_2\right\rangle+\alpha\left|V_2\right\rangle)\left|c^{(+)}\right\rangle+(-\beta\left|H_2\right\rangle+\alpha\left|V_2\right\rangle)\left|c^{(-)}\right\rangle \\ +(\alpha\left|H_2\right\rangle+\beta\left|V_2\right\rangle)\left|d^{(+)}\right\rangle+(-\alpha\left|H_2\right\rangle+\beta\left|V_2\right\rangle)\left|d^{(-)}\right\rangle \end{array}\right] \tag{15}$$

If Bob is told the results of Alice's measurement, he can now rotate the polarization of his photon using a Fresnel rhomb polarization rotator (R_B) to produce the state $\alpha\left|H_2\right\rangle+\beta\left|V_2\right\rangle$. However, because the goal of this experiment is simply to verify that Bob's photon is in the state predicted by eq. 15, R_B is used to rotate the polarization through an angle θ_B that leaves Bob's photon vertically polarized, allowing it to pass through another PBS and hit detector D_B. Alternatively R_B can be set to $\theta_B^{\perp}$ which rotates the photon an extra 90°, causing it to be reflected by the PBS and (in principle) eliminating the possibility of a click at detector D_B. The angle θ_B can be determined by examining the term in eq. 15 that is indicated by Alice's measurement. For instance suppose the Preparer has imprinted the state θ = -120° (using a coordinate system where 0° is horizontal) onto Alice's photon, and that Alice's detector D_{A+} clicks, indicating that the state has collapsed onto the $\left|d^{(+)}\right\rangle$ term. Examining eq. 15, we see that the polarization of Bob's photon is unchanged from Alice's, so he sets θ_B = 30° so that his photon arrives at D_B. Using this scheme, teleportation success can be confirmed by the coincidence count— the number of times two detectors click simultaneously— between detector D_B and one of Alice's detectors (chosen based eq. 15 and knowledge of the values θ and θ_B used to control R and R_B).

The success of the teleportation is quantified by measuring the quantity S = $\left|\langle\phi\left|\phi_{tel}\right\rangle\right|^2$, where $\left|\phi\right\rangle$ ($\left|\phi_{tel}\right\rangle$) is the original (teleported) state. To obtain S, the coincidence rate must be measured when R_B is set to θ_B (call this $I_{\parallel}$) and when R_B is set to $\theta_B^{\perp}$ (call this $I_{\perp}$).

These coincidence rates can be expressed as

$$I_{\parallel} = k\,|\langle\phi\,|\phi_{tel}\rangle|^2 \quad (16a)$$

$$I_{\perp} = k\,|\langle\phi_{\perp}\,|\phi_{tel}\rangle|^2 \quad (16b)$$

The constant "k" has units of s^{-1} and depends on factors which are identical for both coincidence rates such as the detector efficiencies and the rate at which EPR pairs are produced in the BBO crystal. The state $|\phi_{\perp}\rangle$ is used to refer to the fact that when R_B is set to $\theta_B^{\perp}$, the measured coincidence rate is comparing the teleported state to the original state after a 90° rotation. The initial and teleported states are normalized, so $\langle\phi\,|\phi_{tel}\rangle = \sin(\gamma)$ where γ is the angle between the states $|\phi\rangle$ and $|\phi_{tel}\rangle$. Add the two coincidence rates to express S in terms of the coincidence rates:

$$I_{\parallel} + I_{\perp} = k\left(|\sin(\gamma)|^2 + |\sin(\gamma + 90°)|^2\right) = k\left(\sin^2(\gamma) + \cos^2(\gamma)\right) = k \quad (17a)$$

$$S = |\langle\phi\,|\phi_{tel}\rangle|^2 = \frac{I_{\parallel}}{I_{\parallel} + I_{\perp}} \quad (17b)$$

Boschi calculated S for three equally spaced initial polarization states (-120°, 0°, 120°), running the experiment for 10 seconds at a time to collect approximately 500 coincidence counts per run. An evenly weighted average of S in each case yielded S = 0.853 ± 0.012.

How are we to judge the merit of this result? The usual answer is to compare it to the best possible S that could be reached classically (i.e. without using entanglement). Boschi shows that without entanglement, the best possible S is 0.75, so the experimental results break the "classical teleportation limit" by eight standard deviations.

CONCLUSION

The BBCJPW protocol is capable— in principle— of teleporting a photon's polarization over arbitrarily large distances. Experiments have separately demonstrated all the components of the BBCJPW protocol, but no single experiment has reliably teleported an external photon's polarization to another location. It has been shown[10] that this deficiency stems from the fact that it is not possible to perform complete Bell measurements without allowing the quantum systems involved to interact with each other. The Boschi experiment evaded this problem by placing the state to be teleported on the EPR pair itself, but at the cost of being unable to teleport an external photon. Experiments on atoms are easier because they interact more readily than do photons; in 2004 a team led by M. Reibe successfully teleported[11] a superposition of two of the electronic levels of calcium ions. The BBCJPW protocol has also been extended[12] to systems of arbitrary dimensionality and to systems with continuous variables[13], and the latter has been confirmed[14] experimentally.

Quantum teleportation is far from achieving the lofty goal of human transportation set forth by its namesake in science fiction. To date, only certain degrees of freedom of individual particles have been successfully teleported; claiming that a single photon has been teleported in its entirety would require teleporting not only its polarization, but also its frequency, transverse and longitudinal spatial states, and its **k**-vector. Scaling the protocol up to handle molecules, let alone objects containing an Avagadro's number of atoms, is a daunting task not only in terms of performing the multitude of Bell measurements, but also in terms of isolating the object from its environment. In addition, since each Bell measurement produces at least two bits of classical information, transmitting the amount of information necessary to reconstruct an object the size of a human body with today's technology would take longer than the age of the universe. For the forseeable future, though, quantum teleportation can be used to link quantum computers, and the fact that the teleported state literally does not exist between transmitter and receiver can be exploited to send completely secure messages.

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